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Non-linear quantile regression in modeling the diametric growth of cedar (*Cedrela fissilis*)

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ABSTRACT: Understanding the dynamics of tree growth is extremely important to develop effective forest conservation and management strategies. Generally, tree growth is well fitted by non-linear regression models. However, it can commonly present problems caused by heteroscedasticity or possible asymmetry in the distribution of residues. An alternative to overcome this problem is quantile regression, which allows estimates at different quantiles, thus generating a more complete mapping of the development of the forest under study. The objective of this study was to compare the adjustment of the non-linear models Logístico, Gompertz, von Bertalanffy, Brody, Chapman-Richards and Weibull using the least squares method and quantile regression, for data on diameter at breast height (DBH) accumulated over the overtime for 56 trees sampled in native forest using a non-destructive technique. The coefficient of determination, the mean absolute deviation and the Akaike information criterion were used to evaluate the quality of the adjustments and the suitability of the models was verified through residual analysis, with the Brody model being the one that best adhered to the data. All computational analysis was carried out using the free software R.

Key words: Brody model; dendrochronology; growth curve; growth rings

Regressão quantílica não linear na modelagem do crescimento diamétrico de cedro (*Cedrela fissilis*)

RESUMO: Compreender a dinâmica do crescimento das árvores é de extrema importância para desenvolver estratégias eficazes de conservação e manejo das florestas. Geralmente, o crescimento das árvores é bem ajustado por modelos de regressão não linear. Porém, comumente pode apresentar problemas proporcionados pela heterocedasticidade ou possível assimetria na distribuição dos resíduos. Uma alternativa para contornar tal problema é a regressão quantílica, que permite estimativas em diferentes quantis, gerando assim um mapeamento mais completo do desenvolvimento da floresta em estudo. O objetivo deste estudo foi comparar o ajuste dos modelos não lineares Logístico, Gompertz, von Bertalanffy, Brody, Chapman-Richards e Weibull pelo método dos mínimos quadrados e pela regressão quantílica, para os dados do diâmetro a 1,30m do solo (D) acumulado ao longo do tempo para 56 árvores amostradas em floresta nativa com o uso de técnica não destrutiva. Utilizou-se o coeficiente de determinação, o desvio médio absoluto e o critério de informação de Akaike para avaliar a qualidade dos ajustes e verificou-se a adequabilidade dos modelos através da análise residual sendo o modelo Brody o que melhor aderiu aos dados. Toda análise computacional foi realizada utilizando-se o software gratuito e livre R.

Palavras-chave: modelo Brody; dendrocronologia; curva de crescimento; anéis de crescimento

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Introduction

One of the biggest challenges facing us this century is climate change and among the main regulators of the climate on our planet are tropical forests. These forests are responsible for regulating temperature and subtracting carbon from the air, generating rainfall, stabilizing the climate both regionally and globally, among other vital contributions to maintaining life on our planet (Aragão et al., 2022; Portal-Cahuana et al., 2023). Therefore, according to Pumijumnong et al. (2023), understanding the dynamics of tree growth in this type of forest is of paramount importance for developing conservation and management strategies for these ecosystems and predicting future global changes in the carbon cycle and climate.

The study of tree growth rings through dendrochronology has contributed significantly to a better understanding of forest dynamics, making it possible to determine the age of trees and obtain estimates of their growth through non-destructive sampling, which contributes to forest conservation (Aragão et al., 2022; Portal-Cahuana et al., 2023). However, unlike in temperate regions, dendrochronology in the tropics, especially in native forests, has proved to be more complex due to factors such as the high diversity of species, the difficulty of sampling and the lack of distinct growth rings. In this context, it is essential that studies be carried out on appropriate species, among which *Cedrela fissilis* Vell., popularly known as cedar, has stood out with great potential for dendrochronological studies due to the formation of distinct and easily visualized rings (Silva et al., 2019; Pearl et al., 2020; Hammerschlag et al., 2019; Quesada-Román et al., 2022; Pace et al., 2023; Melo Júnior, 2023; Pumijumnong et al., 2023).

Classical non-linear regression has proved very useful for studying growth curves in forestry, because when the measurements of the growth rings accumulate over the years, the distribution has a sigmoidal character, which is well described by non-linear growth models. However, the proper use of this type of regression requires the assumption that the vector of residuals is independent and identically distributed following a normal distribution with zero mean and constant variance (Ribeiro et al., 2018; Jane et al., 2020; Silva et al., 2020). Therefore, after adjusting the regression model, it is extremely important to analyze and diagnose the vector of residuals in order to validate the adjusted model. If the assumptions of homoscedasticity and/or residual independence are violated, these estimators may no longer be efficient, resulting in misleading estimates, causing interpretation problems and an inadequate fit of the model to the data. On the other hand, the normality assumption must be met in order to make inferences about the parameter estimates (Miguez et al., 2018).

However, according to Inga & Valle (2017), many studies in the forestry area have problems related to validating the assumptions of the models used, but these issues are often not addressed or even mentioned (Barbosa et al., 2018; Andrade et al., 2019; Worbes & Schöngart, 2019; Shoda et al., 2020; Pretzsch, 2021). Other studies, such as those by Oliveira et al. (2019), Batista et al. (2020) and Zimbres et al. (2021), suggest using logarithmic transformations as a way of getting around the problems caused by violating the assumptions. However, it is important to note that, according to Mazucheli & Achcar (2002), transformations should be used with caution, as they can lead to the loss of the practical interpretations that the coefficients originally had, as well as difficulties in adequately specifying the errors associated with the model.

Considering the above, it is necessary to study new regression techniques, such as quantile regression, capable of dealing with errors that have non-normal distributions and heterogeneous variances, characteristics often found when studying tree growth. In addition, we can highlight the flexibility of quantile regression, allowing different quantiles of the distribution of the dependent variable to be modeled independently, and its robustness to asymmetric data and the presence of outliers (Koenker, 2005), which makes this type of modeling suitable for dealing with situations commonly found in forestry studies.

With this in mind, the objective of this study was to model the diametric growth of cedar (*Cedrela fissilis*) as a function of age, based on measurements of its annual growth rings with non-destructive samples taken from a native tropical forest and to compare the fits of the non-linear Logistic, Gompertz, von Bertalanffy, Brody, Chapman-Richards and Weibull models using the least squares method and quantile regression.

Materials and Methods

This study used 56 cedar trees (*Cedrela fissilis*) located in a preserved fragment of seasonally dry forest in environmental preservation areas in the municipalities of Montalvânia and Juvenília, in the far north of Minas Gerais, Brazil (latitude 14.50 °S and longitude 44.17 °W).

The data come from the permanent collection of the Dendrochronology Laboratory of the Departamento de Ciências Florestais (DCF) of the Universidade Federal de Lavras (UFLA), Lavras/MG, Brazil. Sampling took place in February and May 2016 using a non-destructive technique with an auger (5.15 mm in diameter and 400 mm long) at breast height (1.30 m). The samples selected reached the pith of the tree and when collecting more than two rays from the same tree, care was taken to ensure that they were diametrically opposed (Barbosa et al., 2018).

Growth ring counting and co-dating were carried out according to the rigorous techniques of dendrochronology detailed in Stokes & Smiley (1996). After co-dating, the widths of the duly dated rings were measured using the LINTAB 6 Scientific measuring system.

In this study, data on the accumulated diameter at 1.30 m from the ground (D) of cedar (*Cedrela fissilis*) was adjusted according to its age, using classical non-linear regression and also quantile non-linear regression. Table 1 shows the non-linear models used in this study, which can also be observed in other forestry research, such as Barbosa et al. (2018) with native tropical forest species, Frühauf et al. (2020) with cedar trees (*Cedrela fissilis*), Gheyret et al. (2021) with subtropical forest species and Nigul et al. (2021) with native and commercial species.

In the equations above, E[Y|+xi] represents the average accumulated D, in cm, x_i the age of the tree, in years, with i ranging from 1 to 154 years, τ refers to the assumed quantile (τ ∈ (0, 1)), Q_τ [Y|x_i] represents the accumulated D, in cm, at quantile τ, α the asymptotic value for the average response, i.e. the expected value for the maximum accumulated D of the trees under study, β the abscissa of the inflection points of the curves, with the exception of the Brody, Chapman-Richards and Weibull models whose curve shapes do not have an inflection point and κ the maturity or precociousness index, the higher it is the less time the tree takes to reach its adult size.

For the quantile function, α(τ) represents the asymptotic value for the quantile response, i.e. the expected value for the maximum accumulated D of the trees under study at the τ quantile, β(τ) the abscissa of the inflection points of the curves at the τ quantile, with the exception of the Brody, Chapman-Richards and Weibull models whose curve shapes do not have an inflection point, and κ(τ) the maturity index at the τ quantile.

After adjusting the models using classical non-linear regression, a residual analysis was carried out to check whether the assumptions of normality, homoscedasticity and residual independence were being met. The Shapiro-Wilk, Breusch-Pagan and Durbin-Watson tests were used to test for normality, homoscedasticity and residual independence, respectively, at the 5% significance level, and graphical analysis was also carried out on the residuals.

Once the model parameters had been estimated, the model that best described the data was compared and selected using the results obtained for Akaike information criterion (Equation 1).

$$\mathrm{AIC} = -2l\left(\hat{\theta}\right) + 2p \tag{1}$$

where, l(θ) is the logarithm of the models maximum likelihood function and p is the number of parameters.

The entire analysis and modeling process involved in this study was carried out using the freely available R statistical software (R Core Team, 2021), using the "nlme", "car", "lmtest", "qpcR" and "quantreg" packages.

Results and Discussion

Initially, an exploratory analysis was carried out to check the behavior of the accumulated D data in cedar trees (*Cedrela fissilis*) over the years. As can be seen in Table 2, the highest value observed was 48.20 cm, which was used as the initial value for estimating the upper horizontal asymptote of the tree growth curve, since the α parameter has this biological interpretation in the non-linear models used (Miguez et al., 2018).

It can be seen that the distribution of the variable under study has a positive asymmetry, since Table 2 shows that the average value of the accumulated D is 16.84 cm, which is greater than the median (14.80 cm) and, in addition, the measure of the asymmetry coefficient found (0.55) is greater than zero and has its absolute value within the interval (0.15; 1], which according to Mattos et al. (2017), indicates a moderate asymmetry to the right.

In order to better visualize the distribution of the data, the histogram of the variable under study was constructed (Figure 1), through which it is possible to observe the concentration of the data in the lowest values, which makes the distribution asymmetrical. Note that the occurrence of

Table 1. Non-linear regression models adjusted to describe the growth curve of cedar (*Cedrela fissilis*) using classical regression and quantile regression.

Models	Classic function	Quantile function
Logistics	$E[Y\|x_i] = \dfrac{\alpha}{1 - e^{\kappa(\beta - x_i)}} + \varepsilon_i$	$Q_\tau[Y\|x_i] = \dfrac{\alpha(\tau)}{1 - e^{\kappa(\tau)(\beta(\tau) - x_i)}} + \varepsilon_i$
Gompertz	$E[Y\|x_i] = \alpha e^{-e^{\kappa(\beta - x_i)}} + \varepsilon_i$	$Q_\tau[Y\|x_i] = \alpha(\tau) e^{-e^{\kappa(\tau)(\beta(\tau) - x_i)}} + \varepsilon_i$
von Bertalanffy	$E[Y\|x_i] = \alpha\left[1 - \dfrac{e^{\kappa(\beta - x_i)}}{3}\right]^3 + \varepsilon_i$	$Q_\tau[Y\|x_i] = \alpha(\tau)\left[1 - \dfrac{e^{\kappa(\tau)(\beta(\tau) - x_i)}}{3}\right]^3 + \varepsilon_i$
Brody	$E[Y\|x_i] = \alpha\left[1 - \beta e^{(-\kappa x_i)}\right] + \varepsilon_i$	$Q_\tau[Y\|x_i] = \alpha(\tau)\left[1 - \beta(\tau) e^{(-\kappa(\tau) x_i)}\right] + \varepsilon_i$
Chapman-Richards	$E[Y\|x_i] = \alpha\left[1 - e^{(-\kappa x_i)}\right]^\beta + \varepsilon_i$	$Q_\tau[Y\|x_i] = \alpha(\tau)\left[1 - e^{(-\kappa(\tau) x_i)}\right]^{\beta(\tau)} + \varepsilon_i$
Weibull	$E[Y\|x_i] = \alpha\left[1 - e^{\left(-\kappa x_i^\beta\right)}\right] + \varepsilon_i$	$Q_\tau[Y\|x_i] = \alpha(\tau)\left[1 - e^{\left(-\kappa(\tau) x_i^{\beta(\tau)}\right)}\right] + \varepsilon_i$

Table 2. Descriptive statistics for the accumulated diameter at 1.30 m above ground (D), in cm, in cedar trees (*Cedrela fissilis*) over the years.

Variable	Minimum	1st quartile	Median	Average	3rd quartile	Maximum	Asymmetry	Coefficient of variation (%)
Cumulative D	0.00	8.34	14.80	16.84	24.84	48.20	0.55	65.24

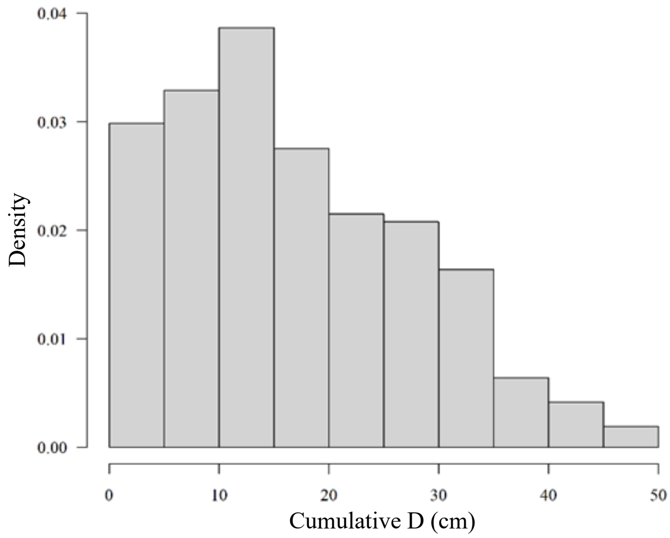

Figure 1. Histogram for the diameter at 1.30 m from the ground (D) accumulated in cedar trees (*Cedrela fissilis*).

high values has a low frequency, which shows a right-tail effect in the distribution of the data, thus confirming that the possible distribution has a positive asymmetry. A similar behavior can be observed in the distribution of accumulated D data in studies such as those carried out by Pereira et al. (2018) and Stepka et al. (2021) on *Cedrela fissilis* trees sampled in the southern region of the country.

After obtaining the estimates of the model parameters using the ordinary least squares method, a residue analysis was carried out to check the validity of the assumptions, i.e. that the residues are independent and identically distributed following a normal distribution with zero mean and constant variance. Table 3 shows the results of the Shapiro-Wilk (normality), Durbin-Watson (independence) and Breusch-Pagan (homoscedasticity) tests.

From the results shown in Table 3, it can be seen that all the models, except the Logistic, met the assumption of residual independence, which can be verified by the Durbin-Watson test (p-value > 0.05). However, the Shapiro-Wilk and Breusch-Pagan tests showed that all the adjusted models presented problems of non-normality and heteroscedasticity in the residual vector (p-value < 0.05).

Figures 2 and 3 show that the graphical analysis corroborated the results obtained by the Breusch-Pagan

Table 3. p-value of the Shapiro-Wilk, Durbin-Watson and Breusch-Pagan tests applied to the residuals of the adjusted models.

Models	Shapiro-Wilk	Durbin-Watson	Breusch-Pagan
Logistics	< 0.001	< 0.001	< 0.001
Gompertz	< 0.001	0.5980	< 0.001
von Bertalanffy	< 0.001	0.6640	< 0.001
Brody	< 0.001	0.1820	< 0.001
Chapman-Richards	< 0.001	0.1740	< 0.001
Weibull	< 0.001	0.1920	< 0.001

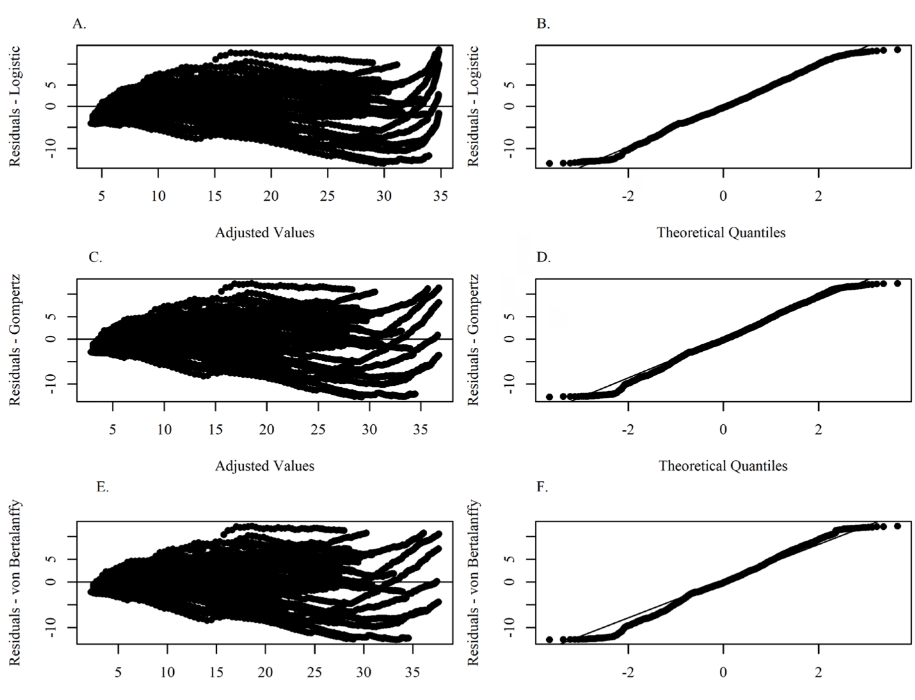

Figure 2. Graphical distribution of residuals for the accumulated diameter at 1.30 m from the ground (D) of cedar trees (*Cedrela fissilis*), where (A), (C) and (E) represent the adjusted values in relation to the residuals and (B), (D) and (F) represent the residual values in relation to the theoretical quantiles for the Logistic, Gompertz and von Bertalanffy models.

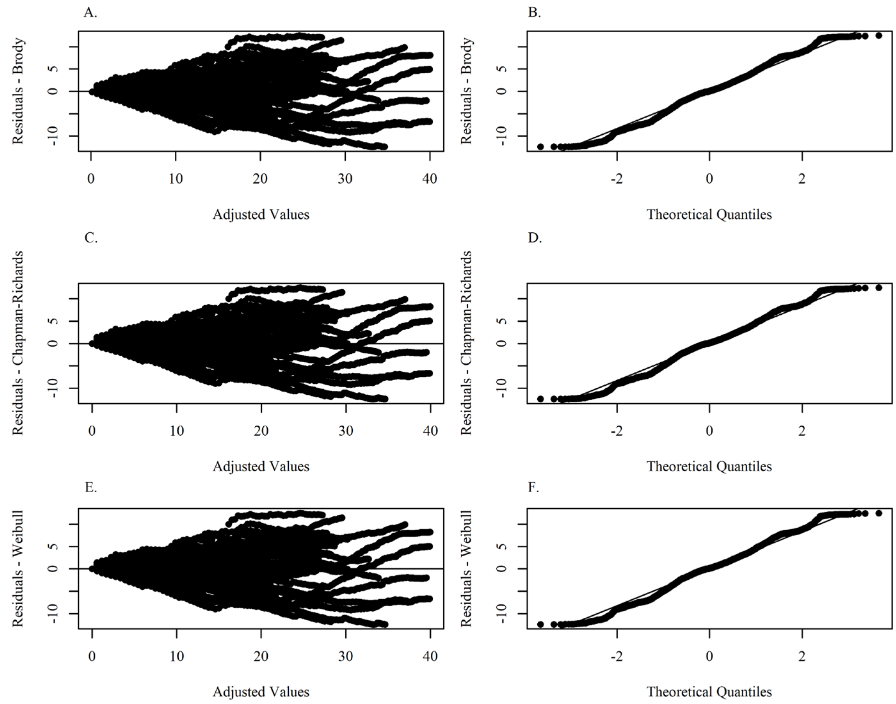

Figure 3. Graphical distribution of the residuals for the accumulated diameter at 1.30 m from the ground (D) of cedar trees (*Cedrela fissilis*), where (A), (C) and (E) represent the adjusted values in relation to the residuals and (B), (D) and (F) represent the residual values in relation to the theoretical quantiles for the Brody, Chapman-Richards and Weibull models.

and Shapiro-Wilk tests in Table 3, indicating heteroscedastic residuals (cone shape) in the graphs of fitted values in relation to the residuals (A, C and E) and non-normality (slight "S" shape) in the graphs of residual values in relation to the theoretical quantiles (B, D and F).

After the residual analysis, it is important to emphasize the importance of the study carried out by Inga & Valle (2017), in which the authors state that the presence of residual heteroscedasticity and non-normality are common in studies in the forestry area, but they are concerned about the fact that many studies are carried out without due attention to residual analysis, which can lead to erroneous conclusions in their research.

Therefore, given the violation of the assumptions of traditional non-linear models, it was decided to adjust the models using quantile regression, which according to Koenker (2005) satisfactorily incorporates the lack of normality and heteroscedasticity in the residue vector. The median ($\tau = 0.50$) and the quantiles $\tau = 0.25$ and $\tau = 0.75$ were used for the tails of the distribution.

Table 4. Estimates of the parameters of the Logistic, Gompertz, von Bertalanffy, Brody, Chapman-Richards and Weibull models adjusted by quantile regression.

Models		α	β	κ	AIC
Logistics	τ = 0.25	28.2893	40.6403	0.0588	24221.18
	τ = 0.50	33.0491	35.6132	0.0624	23700.56
	τ = 0.75	39.0163	35.3150	0.0581	23782.53
Gompertz	τ = 0.25	30.9360	32.7585	0.0329	23919.97
	τ = 0.50	35.6914	27.6954	0.0367	23490.79
	τ = 0.75	43.3281	28.2121	0.0337	23451.75
von Bertalanffy	τ = 0.25	32.4162	27.2448	0.0253	23795.12
	τ = 0.50	37.1546	22.5993	0.0290	23400.28
	τ = 0.75	45.3604	23.0392	0.0266	23330.30
Brody	τ = 0.25	41.1246	1.0000	0.0105	23533.71
	τ = 0.50	46.3680	1.0000	0.0128	23297.42
	τ = 0.75	55.8012	0.9943	0.0125	23213.53
Chapman-Richards	τ = 0.25	46.3231	0.9053	0.0077	23517.23
	τ = 0.50	46.8163	0.9937	0.0126	23297.37
	τ = 0.75	57.8871	0.9535	0.0113	23216.35
Weibull	τ = 0.25	48.1046	0.9181	0.0116	23516.16
	τ = 0.50	45.7757	1.0088	0.0126	23297.70
	τ = 0.75	58.3119	0.9640	0.0136	23217.92

Table 4 shows the parameter estimates and the criteria used to select the model that best fitted the data based on the fit of the Logistic, Gompertz, von Bertalanffy, Brody, Chapman-Richards and Weibull models of the quantile regression at quantiles τ = 0.25, τ = 0.50 and τ = 0.75.

The results shown in Table 4 for the estimates of the α parameter, which represent the asymptotic cumulative D, obtained by the least squares method were very close to those found by the median of the quantile regression (τ = 0.50), which according to Araújo Júnior et al. (2019) may indicate the absence of outliers in the data set, a fact that corroborates the visual analysis of the boxplot constructed for the variable under study (Figure 4).

Another important fact to consider is that Stepka et al. (2021) indicate that cedar trees (*Cedrela fissilis*) can reach 40 to 80 cm of accumulated D, which corroborates the estimates obtained for the asymptotic value (α) by the Brody, Chapman-Richards and Weibull models, which have their values within the aforementioned range for all the quantiles estimated, indicating a good fit, to the detriment of the Logistic, Gompertz and von Bertalanffy models, which presented estimates below the lower limit of the range.

It should also be noted that the estimates of the asymptotic value (α) for the Brody, Chapman-Richards and Weibull models are close to the lower limit of the growth interval for cumulative D indicated by Stepka et al. (2021). However, considering studies such as those by Otalakoski et al. (2021), which indicate a range between 60 and 90 cm for accumulated D in cedar trees, the values estimated in this study are outside this range, a fact that may be justified by the seasonally dry region where the trees were sampled, since according to studies such as those by Venegas-González et al. (2018), the scarcity of water in the area directly interferes with the growth of cedar trees (*Cedrela fissilis*).

Still looking at the estimates of the asymptotic value (α), the Brody model appears to be the one that best fits the data, as it can be seen that the Chapman-Richards model presents very close estimates for the quantiles τ = 0.25 and τ = 0.50 and the Weibull model overestimated the value of α for the quantile τ = 0.25 compared to the median (τ = 0.50).

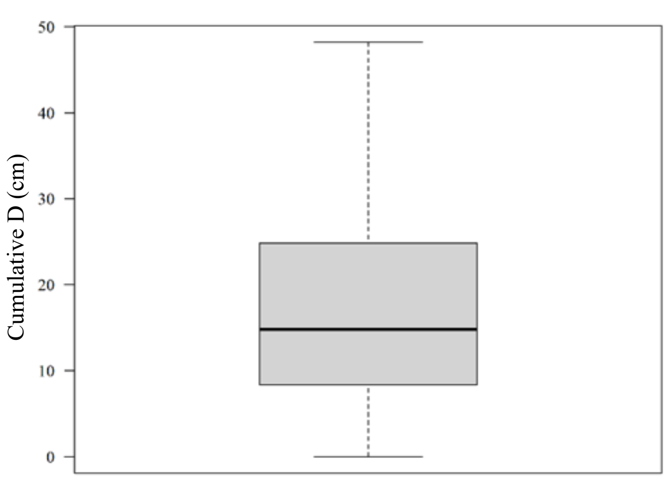

Figure 4. Boxplot for the accumulated diameter at 1.30 m from the ground (D), in cm, in cedar trees (*Cedrela fissilis*).

The κ parameter represents the maturity index, i.e. the lower its value, the longer it takes the object under study to reach the asymptotic accumulated D. Thus, it can be seen that for the adjusted models, the τ = 0.25 quantile showed the lowest estimates, indicating that cedar trees with the lowest accumulated D take longer to reach maturity compared to medium (τ = 0.50) and larger (τ = 0.75) trees.

The criteria used to check the quality of the adjustments will be compared in the same quantiles, because according to Puiatti et al. (2018), comparing measures of quality of adjustment in different quantiles for the same data is inconsistent.

Looking at the results presented in Table 4 for the quality of fit criterion, it can be seen that the fits of the Brody, Chapman-Richards and Weibull models were very similar to each other and proved to be superior to the other models tested, as they obtained lower AIC values compared to the other models.

Figure 5 shows the graphs of the fits of the Logistic (A), Gompertz (B), von Bertalanffy (C), Brody (D), Chapman-

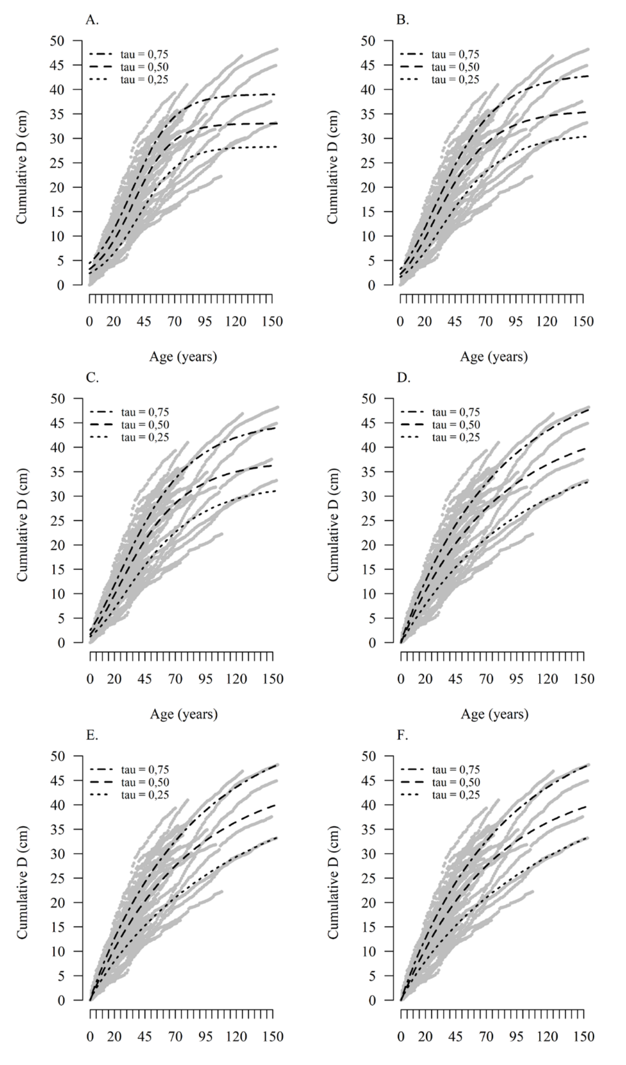

Figure 5. Adjustment of the Logistic (A), Gompertz (B), von Bertalanffy (C), Brody (D), Chapman-Richards (E) and Weibull (F) models to the accumulated diameter data at 1.30 m from the ground (D), in cm, as a function of age, in years, of cedar trees (*Cedrela fissilis*).

Richards (E) and Weibull (F) models to the accumulated D data of cedar trees (*Cedrela fissilis*) over the years.

Visual analysis of the graphs suggests that the Logistic (A), Gompertz (B) and von Bertalanffy (C) models did not adhere well to the data, overestimating the initial observations and underestimating the final ones. The Brody (D), Chapman-Richards (E) and Weibull (F) models showed good and similar fits, thus confirming the results found by the criteria used to assess the quality of the fits.

As these models do not show an inflection point, an exponential trend can still be observed in the growth of these trees, which may be due to the fact that this population does not yet contain individuals who have reached the final stage of their growth cycle, as the sigmoidal trend can be seen when at least some individuals are nearing the end of their growth cycle (Scolforo, 2006).

Conclusions

Quantile non-linear regression proved to be suitable for describing the accumulated D over time of cedar trees (*Cedrela fissilis*) in native forest with asymmetric and heteroscedastic residues, and can be used as an alternative to aid growth studies, without disregarding the essential conventional regression models and their extensions.

The Brody model for quantile regression was the one that best fit the data, thus indicating an exponential character in the growth of the forest under study.

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Compliance with Ethical Standards

Author contributions: Conceptualization: JAM; Data curation: ACF; Formal analysis: KPL; Investigation: GAP, ACMCB; Methodology: TJF; Project administration: JAM; Software: KPL; Supervision: JAM; Writing – original draft: ACF;

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